

**NASA
Technical
Memorandum**

NASA TM -82582

**IRRADIANCE FROM STARS BASED ON
BLACKBODY THEORY**

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June 1984



National Aeronautics and
Space Administration

George C. Marshall Space Flight Center

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TECHNICAL MEMORANDUM

IRRADIANCE FROM STARS BASED ON BLACKBODY THEORY

INTRODUCTION

This report describes a computer program which calculates a theoretical blackbody curve for a star when given only the star's temperature and visual magnitude. The program was written to produce approximate stellar spectra to be used in the design of the Space Telescope Fine Guidance System (FGS). In addition to the blackbody curve, the program calculates the number of photons that would be received from the star by the Space Telescope per 25 msec interval in wavelength increments of 0.2 microns. The equations used for the computation work best for stars that emit primarily in the range from near infrared to ultraviolet. Cooler stars may not give accurate results.

DESCRIPTION OF PROGRAM CALCULATIONS

This computer program is designed to give data for a blackbody curve and a count of photons emitted by the star. The only inputs are the temperature and visual magnitude of the star being used.

The first step in the program is to find the visible irradiance for a star $[I(m_v)]$ using equation (1).

$$m_v = -2.5 \log_{10} [I(m_v)/I_0] \quad (1)^1$$

Where

I_0 = irradiance of zero magnitude star

m_v = visible magnitude

Equation (1) easily converts to equation (2) with $I(m_v)$ dependent on I_0 and m_v .

$$I(m_v) = I_0 \times 10^{-(0.4m_v)} \text{ Watts cm}^{-2} \quad (2)$$

To find the blackbody visible response fraction $[\eta_e(T)]$ equation (3) is used.

$$\eta_e(T) = \frac{\int W_{\lambda}(T) S_{e\lambda} d\lambda}{\int W_{\lambda}(T) d\lambda} \quad (3)^1$$

Where

S_e = the human eye response²

$W_\lambda(T)$ = the Planck function, equation (4)

T = temperature in degrees Kelvin

λ = wavelength

and the Planck function wavelength (λ) is given by,

$$W_\lambda(T) = \frac{2\pi c^2 h}{\lambda^5 (e^{hc/\lambda kT} - 1)} \text{ Watts/cm}^2/\mu \quad (4)^3$$

where

c = speed of light

k = Planck's constant

h = Boltzman's constant

The top integral in equation (3) is calculated in the program over lengths of 0.34 to 0.82 with a $d\lambda$ (delta wavelength) of 0.02 microns. The bottom integral is the Stefan Boltzman function which is equal to

$$5.679 \times 10^{-12} \times T^4 \text{ watts/cm}^2 \quad (5)^1$$

To find the maximum value of the Planck function [$W_{\lambda \text{ max}}(T)$] equation (6) is used.

$$W_{\lambda \text{ max}}(T) = 1.25 \times 10^{-15} \times T^5 \text{ watts/cm}^2/\mu \quad (6)^1$$

Using equations (2), (3), (5) and (6), the peak of the blackbody curve becomes equation (7).

$$H_{\lambda \text{ peak}} = \frac{I(m_v)}{\eta_e(T)} \times \frac{W_{\lambda \text{ Max}}(T)}{\int W_\lambda(T) d\lambda} \text{ watts/cm}^2/\mu \quad (7)^1$$

From the value for $H_{\lambda\text{peak}}$ and using equation (8),

$$H(\lambda, T) = \frac{0.2900}{(\lambda T)^5 (e^{1.438/\lambda T} - 1)} \times H_{\lambda\text{peak}} \text{ watts/cm}^2/\mu \quad (8)^1$$

values for the blackbody curve are calculated for each (wavelength) used. This data then can be used to create a plot for the blackbody star.

To find the amount of watts/ μ received, the blackbody data was multiplied by a collecting aperture (A). The aperture used in the sample is from the Space Telescope. To give an approximation in watts over an area around the wavelength increments the watts/ μ is multiplied by 0.02 microns.

$$H_{\text{st}}(\lambda, T) = H(\lambda, T) \times A \times 0.02 \text{ watts} \quad (9)$$

Watts are divided by hc/λ to give photons. As is shown in equation (10).

$$H_p(\lambda, T) = \frac{H_{\text{st}}(\lambda, T)}{hc/\lambda} \text{ photons/sec} \quad (10)$$

CONCLUSION

By the use of this computer program, theoretical values are created for a blackbody curve. These values are calculated from a star's visual magnitude and may be used to produce photon counts at certain wavelengths for use in many applications. These applications include throughput analysis of lenses and mirrors where efficiency is important.

The blackbody curve created with this program is only applicable outside the atmosphere. For operation inside the atmosphere, values of spectral absorption should be applied. The values calculated are assumed to be from a star which behaves as a "Planckian Emitter." There is evidence that while many stars behave this way, some do not. This is especially true for cooler stars.

CONSTANTS

$$*I_o = 3.1 \times 10^{-13} \text{ watts cm}^2$$

$$c = 2.99793 \times 10^{10} \text{ cm/sec}$$

$$h = 6.626176 \times 10^{-34} \text{ J.S}$$

$$k = 1.380662 \times 10^{-23} \text{ J/K}^\circ$$

$$\pi = 3.1415926$$

$$A = 3.89 \times 10^4 \text{ cm}^2 \text{ for Space Telescope}$$

*Allen's Astrophysical Journal states that $I_o = 3.6 \times 10^{-13} \text{ watts cm}^2$, but to be conservative the Applied Optics value of $3.1 \times 10^{-13} \text{ watts cm}^2$ is used.

EXAMPLE RUN OF PROGRAM

This program is currently being used in a throughput analysis program for the Fine Guidance System of the Space Telescope. The values for $H(\lambda, T)$ are normalized (Fig. 1) and multiplied with the throughput value of the other surfaces (Fig. 2), producing a sample of the amount of photons that will reach the optical detectors that produce a signal for fine pointing of the Space Telescope. Two example runs of the program show the number of photons emitted by the star.

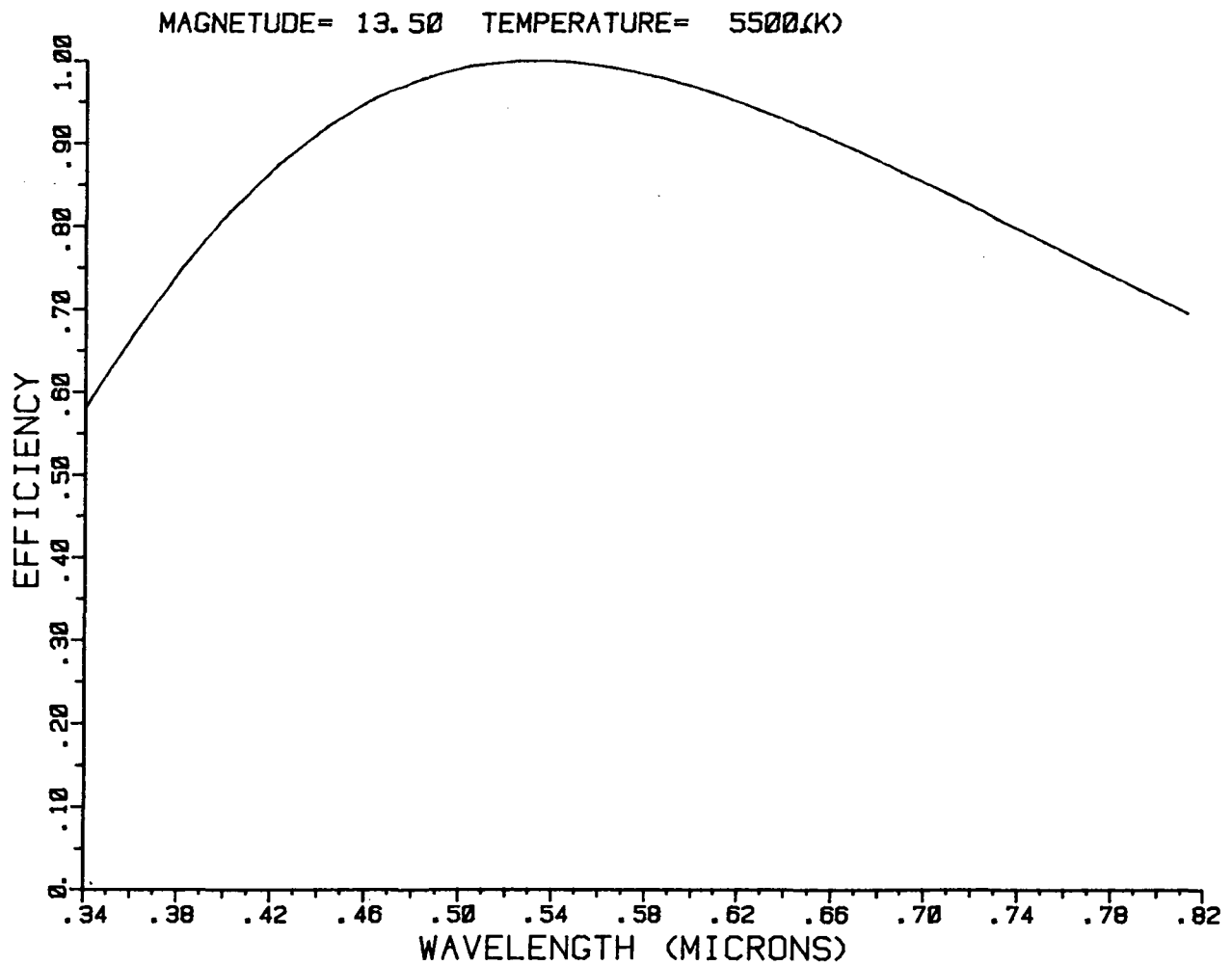
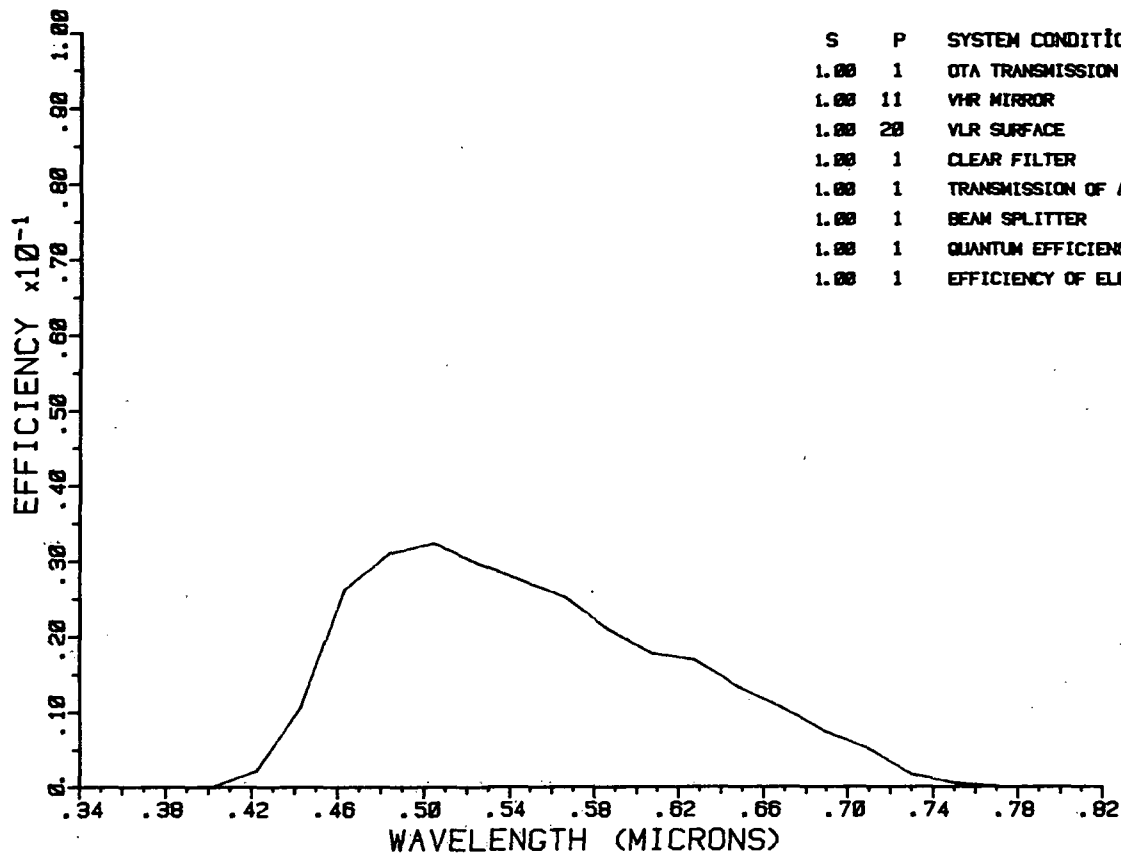


Figure 1. Normalized blackbody curve of a star.



S	P	SYSTEM CONDITION
1.00	1	OTA TRANSMISSION - IRD SPEC
1.00	11	VHR MIRROR
1.00	20	VLR SURFACE
1.00	1	CLEAR FILTER
1.00	1	TRANSMISSION OF ALL GLASS AND MONOSIL
1.00	1	BEAM SPLITTER
1.00	1	QUANTUM EFFICIENCY OF DETECTOR
1.00	1	EFFICIENCY OF ELECTRONICS

Figure 2. OTA-FGS throughput curve.

SAMPLE RUN OF PROGRAM BB DATAPROG

CONTROL COMMANDS

!SET F:20/EYERESP

!SET F:TT ME:DRC:BIN

START OF PROGRAM

!START BB DATAPROG

IN THIS BLACKBODY DATA PROGRAM, YOU WILL
ENTER A MAGNETUDE AND TEMPERATURE(K)
AND THE PROGRAM WILL COMPUTE THE PHOTONS PER
WAVELENGTH.

ENTER THE MAGNETUDE AND TEMPERATURE(K).
?13.5,5500

WAVELENGTH=	..340	H(LAMBDA)=	.685511E-17	PHOTONS=	228.
WAVELENGTH=	..360	H(LAMBDA)=	.789838E-17	PHOTONS=	278.
WAVELENGTH=	..380	H(LAMBDA)=	.883652E-17	PHOTONS=	329.
WAVELENGTH=	..400	H(LAMBDA)=	.964913E-17	PHOTONS=	378.
WAVELENGTH=	..420	H(LAMBDA)=	.103264E-16	PHOTONS=	425.
WAVELENGTH=	..440	H(LAMBDA)=	.108668E-16	PHOTONS=	468.
WAVELENGTH=	..460	H(LAMBDA)=	.112750E-16	PHOTONS=	508.
WAVELENGTH=	..480	H(LAMBDA)=	.115597E-16	PHOTONS=	543.
WAVELENGTH=	..500	H(LAMBDA)=	.117323E-16	PHOTONS=	574.
WAVELENGTH=	..520	H(LAMBDA)=	.118055E-16	PHOTONS=	601.
WAVELENGTH=	..540	H(LAMBDA)=	.117921E-16	PHOTONS=	623.
WAVELENGTH=	..560	H(LAMBDA)=	.117050E-16	PHOTONS=	642.
WAVELENGTH=	..580	H(LAMBDA)=	.115561E-16	PHOTONS=	656.
WAVELENGTH=	..600	H(LAMBDA)=	.113563E-16	PHOTONS=	667.
WAVELENGTH=	..620	H(LAMBDA)=	.111156E-16	PHOTONS=	675.
WAVELENGTH=	..640	H(LAMBDA)=	.108428E-16	PHOTONS=	679.
WAVELENGTH=	..660	H(LAMBDA)=	.105454E-16	PHOTONS=	681.
WAVELENGTH=	..680	H(LAMBDA)=	.102302E-16	PHOTONS=	681.
WAVELENGTH=	..700	H(LAMBDA)=	.990262E-17	PHOTONS=	679.
WAVELENGTH=	..720	H(LAMBDA)=	.956753E-17	PHOTONS=	674.
WAVELENGTH=	..740	H(LAMBDA)=	.922884E-17	PHOTONS=	669.
WAVELENGTH=	..760	H(LAMBDA)=	.888981E-17	PHOTONS=	662.
WAVELENGTH=	..780	H(LAMBDA)=	.855313E-17	PHOTONS=	653.
WAVELENGTH=	..800	H(LAMBDA)=	.822094E-17	PHOTONS=	644.
WAVELENGTH=	..820	H(LAMBDA)=	.789495E-17	PHOTONS=	634.

STOP END OF PROGRAM

SAMPLE RUN OF PROGRAM BB DATAPROG

Run #2

CONTROL COMMANDS

!SET F:20/EYERESP

!SET F:TT ME:DRC:BIN

START OF PROGRAM

!START BB DATAPROG

IN THIS BLACKBODY DATA PROGRAM, YOU WILL
ENTER A MAGNETUDE AND TEMPERATURE(K)
AND THE PROGRAM WILL COMPUTE THE PHOTONS PER
WAVELENGTH.

ENTER THE MAGNETUDE AND TEMPERATURE(K).
?14.5,5500

WAVELENGTH=	.340	H(LAMBDA)=	.272908E-17	PHOTONS=	91.
WAVELENGTH=	.360	H(LAMBDA)=	.314441E-17	PHOTONS=	111.
WAVELENGTH=	.380	H(LAMBDA)=	.351789E-17	PHOTONS=	131.
WAVELENGTH=	.400	H(LAMBDA)=	.384140E-17	PHOTONS=	150.
WAVELENGTH=	.420	H(LAMBDA)=	.411103E-17	PHOTONS=	169.
WAVELENGTH=	.440	H(LAMBDA)=	.432617E-17	PHOTONS=	186.
WAVELENGTH=	.460	H(LAMBDA)=	.448367E-17	PHOTONS=	202.
WAVELENGTH=	.480	H(LAMBDA)=	.460201E-17	PHOTONS=	216.
WAVELENGTH=	.500	H(LAMBDA)=	.467073E-17	PHOTONS=	229.
WAVELENGTH=	.520	H(LAMBDA)=	.469986E-17	PHOTONS=	239.
WAVELENGTH=	.540	H(LAMBDA)=	.469455E-17	PHOTONS=	248.
WAVELENGTH=	.560	H(LAMBDA)=	.465987E-17	PHOTONS=	256.
WAVELENGTH=	.580	H(LAMBDA)=	.460057E-17	PHOTONS=	261.
WAVELENGTH=	.600	H(LAMBDA)=	.452105E-17	PHOTONS=	266.
WAVELENGTH=	.620	H(LAMBDA)=	.442523E-17	PHOTONS=	269.
WAVELENGTH=	.640	H(LAMBDA)=	.431660E-17	PHOTONS=	270.
WAVELENGTH=	.660	H(LAMBDA)=	.419823E-17	PHOTONS=	271.
WAVELENGTH=	.680	H(LAMBDA)=	.407272E-17	PHOTONS=	271.
WAVELENGTH=	.700	H(LAMBDA)=	.394232E-17	PHOTONS=	270.
WAVELENGTH=	.720	H(LAMBDA)=	.380892E-17	PHOTONS=	269.
WAVELENGTH=	.740	H(LAMBDA)=	.367408E-17	PHOTONS=	266.
WAVELENGTH=	.760	H(LAMBDA)=	.353911E-17	PHOTONS=	263.
WAVELENGTH=	.780	H(LAMBDA)=	.340507E-17	PHOTONS=	260.
WAVELENGTH=	.800	H(LAMBDA)=	.327283E-17	PHOTONS=	256.
WAVELENGTH=	.820	H(LAMBDA)=	.314305E-17	PHOTONS=	252.

STOP END OF PROGRAM

*TY1-42

```
1.000 C ***** BLACKBODY DATA PROGRAM *****
2.000 C
3.000 C
4.000 C THIS PROGRAM IS DESIGNED TO CREATE DATA FOR
5.000 C A BLACK BODY CURVE FROM A GIVEN TEMPERATURE
6.000 C AND A GIVEN MAGNETUDE
7.000 C WRITTEN BY WILLIAM JACOBS
8.000 C
9.000 DIMENSION S(100),W(100),R(100),PHOTON(100)
10.000 DIMENSION HST(100),H(100)
11.000 10 WRITE(102,20)
12.000 20 FORMAT('IN THIS BLACKBODY DATA PROGRAM, YOU WILL',/,
13.000 '$ENTER A MAGNETUDE AND TEMPERATURE(K)',/,
14.000 '$AND THE PROGRAM WILL COMPUTE THE PHOTONS PER',/,
15.000 '$WAVELENGTH. ',/,
16.000 '$//, 'ENTER THE MAGNETUDE AND TEMPERATURE(K).')
17.000 READ(101,30)SMAG,TEMP
18.000 30 FORMAT(2G)
19.000 C
20.000 C CALCULATE VALUE FOR VISIBLE IRRADIANCE
21.000 C
22.000 WFC=3.1E-13*(10**(-.4*SMAG))
23.000 N=0
24.000 VISUAL=0
25.000 DO 50 N=1,24
26.000 C
27.000 C INPUT VALUES OF EYERESPONCE FOR EACH WAVELENGTH
28.000 C
29.000 READ(20,40)S(N)
30.000 40 FORMAT(1G)
31.000 50 CONTINUE
32.000 N=N+1
33.000 DO 60 WAVLEN=.34,.82,.02
34.000 N=N+1
35.000 C
36.000 C CONVERT MICRONS TO CENTIMETERS
37.000 C
38.000 SD=WAVLEN*1E-4
39.000 C
40.000 C CALCULATION OF PLANCK FUNCTION
41.000 C
42.000 W(N)=3.74185E-12/((SD**5)*(EXP(1.4388/(SD*TEMP))-1))
```

*

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*TY43-84
43.000 C
44.000 C      INTEGRATE EYE RESPONSE DATA TIMES PLANCK FUNCTION
45.000 C
46.000      VISUAL=VISUAL+W(N)*S(N)*2E-6
47.000      60 CONTINUE
48.000      TOT=5.679E-12*TEMP**4
49.000 C
50.000 C      VALUE FOR VISIBLE RESPONSE FRACTION
51.000 C
52.000      EFF=VISUAL/TOT
53.000      N=0
54.000      DO 80 WAVLEN=.34,.82,.02
55.000      N=N+1
56.000 C
57.000 C      LOCATE PEAK OF BLACKBODY CURVE
58.000 C
59.000      HPEAK=(WPC/EFF)*((1.29E-15*TEMP**5)/(5.679E-12*TEMP**4))
60.000      SA=EXP(1.4380/((WAVLEN*1E-4)*TEMP))-1
61.000 C
62.000 C      CALCULATE VALUES FOR BLACKBODY CURVE
63.000 C      AT EACH WAVELENGTH INCREMENT
64.000 C
65.000      H(N)=(.2900/((((WAVLEN*1E-4)*TEMP)**5)*SA))*HPEAK
66.000 C
67.000 C      AREA FOR SPACE TELESCOPE EQUALS 3.89E4 CM**2
68.000 C
69.000      HST(N)=H(N)*3.89E4*.02
70.000 C
71.000 C      CALCULATE NUMBER OF PHOTONS FOR EACH WAVELENGTH
72.000 C
73.000      PHOTON(N)=HST(N)/(1.9864776E-23/(WAVLEN*1E-4))
74.000 C
75.000 C      CONVERT PHOTONS/SEC TO PHOTONS/25MILLISEC
76.000 C
77.000      PHOTON(N)=PHOTON(N)*.025
78.000 C
79.000      WRITE(102,70)WAVLEN,H(N),PHOTON(N)
80.000      70 FORMAT(/,1X,'WAVELENGTH= ',F6.3,3X,'H(LAMBDA)= ',1G,
81.000      $3X,'PHOTONS= ',F6.0)
82.000      80 CONTINUE
83.000      STOP 'END OF PROGRAM'
84.000      END

```

*

Variables in Program and Their Meaning

SMAG	Visual magnitude
Temp	Temperature of star
WPC	Visible irradiance
S(N)	EYE response data
W(N)	Planck function
Visual	Integration value of visible spectrum
TOT	Stefan Boltzman function
EFF	Blackbody visible response fraction
HPEAK	Peak of blackbody curve
H(N)	Values for blackbody curve
Hst(N)	Values represented in watts
Photon (N)	Number of photons at each wavelength

LIST OF EYE RESPONSE DATA FOR PROGRAM

!E EYERESP
EDIT HERE
#TY

.340	0.0
.360	0.0
.380	0.00004
.400	0.0004
.420	0.0040
.440	0.023
.460	0.060
.480	0.139
.500	0.323
.520	0.710
.540	0.954
.560	0.995
.580	0.870
.600	0.631
.620	0.381
.640	0.175
.660	0.061
.680	0.017
.700	0.0041
.720	0.0
.740	0.0
.760	0.0
.780	0.0
.800	0.0
.820	0.0

*

REFERENCES


1. Ramsey, R. C.: Spectral Irradiance from Stars and Planets Above the Atmosphere, from 0.1 to 100.0 microns. Applied Optics, Vol. 1, No. 4, July 1962, pp. 465-471..
2. RCA Electro-Optics Handbook Technical Series EOH-11, page 54.
3. RCA Electro-Optics Handbook Technical Series EOH-11, page 35.

APPROVAL

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The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.



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Director, Information and Electronic
Systems Laboratory